Toward a Semiotics of Motion

Nicolas Mansard
Applicative context
Definition of the problem

- Find a path
- a trajectory
- a control

that makes the robot accomplished a given task
Direct geometry

Robot configuration: \( q \)
Robot end-effector position: \( h(q) \)

\[
h(q) = \begin{bmatrix}
    l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\
    l_1 \sin(q_1) + l_2 \sin(q_1 + q_2)
\end{bmatrix}
\]

Find \( c_1, s_1, c_2, s_2 \)

s.t.

\[
\begin{align*}
    l_1 c_1 + l_2 c_2 &= h_x^* \\
    l_1 s_1 + l_2 s_2 &= h_y^* \\
    c_1^2 + s_1^2 &= 1, \quad c_2^2 + s_2^2 &= 1
\end{align*}
\]
Direct dynamics

Robot configuration: \( q \)

Robot end-effector position: \( h(q) \)

\[
    h(q) = \begin{bmatrix}
        l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\
        l_1 \sin(q_1) + l_2 \sin(q_1 + q_2)
    \end{bmatrix}
\]

Torque control input: \( \tau \)

\[
    \ddot{q} = A(q)^{-1} \tau - b(q, \dot{q})
\]

Optimal control problem:

Find \( \tau(t) \)

s.t. \( q(0) = q_0, h(q(1)) = h^* \)

\[
    \ddot{q} = A(q)^{-1} \tau - b(q, \dot{q})
\]
Optimal control problem

Find $\tau(t)$

subject to $q(0) = q_0$, $h(q(1)) = h^*$

$\ddot{q} = A(q)^{-1} \tau - b(q, \dot{q})$
Optimal control problem

[Koch, Mansard, Souères, Mombaur, submitted IEEE ICRA’14]
Optimal control problem

- Technological gap to bridge
  - 3 hours for the stepping motion
  - 10 ms needed on the robot

- Robotics
  - Trade off between models and methods

\[ \begin{array}{cccc}
\text{joint} & \text{joint} & \text{joint} & \text{joint} \\
\text{joint} & \text{joint} & \text{joint} & \text{joint} \\
\vdots & \vdots & \vdots & \vdots \\
\text{joint} & \text{joint} & \text{joint} & \text{joint} \\
\text{center} & \text{center} & \cdots & \text{center} \\
\end{array} \]
Task-function approach

- The task function

A regular function of the configuration

\[ e: q, \Omega \rightarrow e(q, \Omega) \]

- The task Jacobian

The derivative with respect to the configuration

\[ J = \frac{\partial e}{\partial q}: \mathcal{J}_0(q) \]

- Task reference dynamics

A vector field in the task image

\[ e(q) \rightarrow \dot{e}^* \]
Task-function approach

- Easy motion specification
- Reusability - versatility
- Deformation of the motion
- Sensor feedback

\[ M\ddot{e} = -D\dot{e} - Ke + f \]

\[ \dot{e} = 0 \]

\[ \min \dddot{e} \]

\[ c - \frac{1}{\omega^2} \dot{e} \in S \]

\[ \dot{e} = -Ke \]

Courtesy from A. Billard
Converter from discrete “symbolic” orders
... to continuous “numeric” controls
Proto symbol

[Walk to the ball
Grasp it

Walk to the bin
Trash the ball

When close enough
Reaching failed
Reaching succeeded
When close enough

[Mansard, Stasse, Yokoi, Chaumette, IEEE ICRA'07]
Semiotics of motion

- Studying motion symbols and their interpretation as physical motion
  - A meaningful bridge between symbolic reasoning and numerical execution
  - Simplify the high level processes by encapsulating a part of the numerical complexity

- Work structure
  - Axis 1: construction of a motion vocabulary
  - Axis 2: explore the corresponding semantics
Outline

Axis 1
1. Hierarchical problems
2. Inverse dynamics and examples

Axis 2
1. Task sequence from observation
2. Task sequence from planning

Conclusion
   Toward a semiotics of movement
Composition of tasks

- Kinematic singularity
  - Inherent to the task
  - Reasonably well identified and treated

- Algorithmic singularity
  - Conflict between tasks

\[
\min_x \|A_1 x - b_1\|^2 + \|A_2 x - b_2\|^2 \\
\text{s.t.} \quad A_0 x = b_0
\]
Hierarchical problem definition

- Weighted task sum:
  \[ e = \begin{bmatrix} \alpha_1 e_1 \\ \vdots \\ \alpha_n e_n \end{bmatrix} \]
  \[ \alpha_{i+1} / \alpha_i \rightarrow 0 \]

- Hierarchy at the limit:
  \[ \alpha_i = \varepsilon^{i-1} \]

\[ \lim_{\varepsilon \rightarrow 0} q_{\text{weight}} = q_{\text{stack}} \]
Hierarchical Quadratic Program

- … in 10 clicks [Kanoun09]

\[ w_2^* = \min \| w_p \|^2 \]

\[ A_1 x \leq b_1 - w_1^* \]

\[ A_2 x \leq b_2 - w_2^* \]

\[ \vdots \]

\[ A_{p-1} x \leq b_{p-1} - w_{p-1}^* \]

\[ A_p x \leq b_p - w_p \]
HQP resolution

- Primal active search
  - “All the constraints are satisfied or active”
  - First phase (feasible point) included
  - The complexity of the dual is higher

- Null-space resolution
  - “Flat” matrices
  - Equivalent to the classical “Siciliano” resolution

[Escande, Mansard, Wieber, IEEE ICRA’10]
HQP results

Experiment A

The robot tracks a moving ball with its right hand, while trying to keep it in its FOV and optimize its balance. When the set of tasks becomes unfeasible, the hierarchy is followed...

\[ \text{joint-limits} < \text{contact support} < \text{grasp} < \text{fov} < \text{balance} \]
Empirical complexity

**SVD + projection**

**HCOD**

- Computation time (s)
- Levels

- Values:
  - 0.010
  - 0.008
  - 0.006
  - 0.004
  - 0.002
  - 0

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Nicolas MANSARD

Gepetto Team

20/50
Summary

- Hierarchical solver
  - Not more complex to implement
  - Slightly more efficient
  - No off-the-shelf implementation

- One step toward automatic regularization

\[
\text{lsprox } ||w_1||^2 + \eta ||x||^2 < ||w_2||^2 + \eta ||x||^2 < \ldots < ||w_p||^2 + \eta ||x||^2 \\
A_1 x \leq b_1 - w_1^* \\
\vdots \\
A_p x \leq b_p - w_p
\]
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Operational space inverse dynamics

- References
  \[ J_1 \ddot{q} + J_1 \dot{q} = \ddot{e}_1^* \]

- Variables (control)
  - Joint torques: \( \tau \)
  - External forces: \( f \)
  - Joint acceleration: \( \ddot{q} \)

- Constraints
  - Dynamic equation
    \[ A \ddot{q} + b = S^T \tau + J^T f_{\text{ext}} \]
  - Contact constraint
    \[ J_c \ddot{q} + J_c \dot{q} = 0 \]
    \[ f_\perp > 0 \]

[Saab, Mansard, Fourquet, Souères, IEEE ICRA'11]
Decoupling ill conditioning

Accelerations

Feasible movements

Numerically feasible movements

forces

Quasi bilateral contact

Unilateral contact

\( \alpha \approx 0 \)

\( \alpha \approx 180 \)

[Mansard, IEEE ICRA’12]
Multiple contact

[Saab, Ramos, Keith, Mansard, Fourquet, Souères, IEEE TRO’12]
Robust walk

[Ramos, Mansard, Stasse, Souères, IEEE Humanoid’12]
Toward model predictive control

\[
\min_{\tau_0 \ldots \tau_T} e_T(q_T)^2 + \int_0^T e(q_t)^2 + r(\tau_t)^2 dt
\]

Inverse dynamics
Residual of \(e_T\)
Residual of \(e_t\)
Inverse dynamics
\(e_T\)
\(e_t\)
\(r_t\)
Toward model predictive control

Control-limited Differential Dynamic Programming

Yuval Tassa, Nicolas Mansard, Emo Todorov

[Tassa, Mansard, Todorov, submitted IEEE ICRA’14]
[Mansard, Stasse, Evrard, Kheddar, IEEE ICAR'09]
Robot @ CWE

[Stasse, Evrard, Mansard et al. IEEE IROS’09]
Dance with HRP-2

[Ramos, Mansard, Stasse et al. submitted IEEE RAM]
Task program
Planning
Imitation

Symbolic controller

Solver

Complex and uncertain world

push
remove
swap

\[
\begin{align*}
& e_n \\
& e_i \\
& e_{i-1} \\
& e_1
\end{align*}
\]
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Conclusion
Toward a semiotics of movement
Understand to replicate

Geometrical extraction

Direct replay

Tracking center of mass and hands, posture last

[Ramos, Saab, Hak, Mansard, IEEE Humanoid’11]
Overall process

- Demonstrator SOT: com, gaze, rhand
- Selection + numerical values
- Possible tasks: task1, task2, task3, task4, task5
- Observation SOT: com, gaze, rhand
- Joint trajectories
Task selection

- Fit the observed trajectory by optimization

\[ x^* = \arg \min_x \| \mathbf{p}^*(t) - \mathbf{p}_x(t) \| \]

- The residue value discriminates between tasks

[Hak, Mansard, Stasse, Laumond, IEEE TSMC, 2012]
Tasks projection

[Hak, Mansard, Stasse, Laumond, IEEE TSMC, 2012]
Example

Grab one ball

Grab two balls

Right hand
Center of mass
Two feet

Right hand
Left hand
Center of mass
Two feet
A task is a motion lexem

To **specify** a motion to be performed …

… but also to **describe** an observed motion
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Context

From François Chaumette
Trajectory as a task sequence

- The task implicitly defines a unique trajectory by integration from an initial point
- The robustness domain is a tube around the trajectory

Mission planning

Tasks planning

Optimized tasks sequencing

Stack of Tasks

Control Loop

Control Loop
Task warping in a nutshell

- For each task:
  - Insertion and removal times
  - Some numerical parameters (gain, intermediate position)
- Numerical solver
  - Chooses the times and parameters
- Dynamics simulator
  - Integrate the trajectory and check the feasibility

[Keith, Mansard, Miossec, Kheddar, IEEE IROS’09]
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Absorb as much as possible the numerical complexity from the lowest level

A control that would be … … whole-body … dynamics-consistent … model-predictive … sensor-based
Motion semantic

- Unifying planning and control
- From folded space to symbolic plan
- Task-based trajectory definition
- Using the synergies

Discovering and encoding the prior that makes the local motion generator converge
Member of a team ...